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SPACE ELEVATOR CLIMBER DYNAMICS ANALYSIS AND CLIMB FREQUENCY OPTIMISATION

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Abstract

The paper describes a spreadsheet-based analysis of the motion of a climber ascending an Earth space elevator tether. The tether is represented by elements of varying lengths, each of mean cross-sectional area based on a taper ratio equation from earlier studies. The tether tension force is calculated in each element based on gravity and centrifugal forces plus the tension force in the element below. A climber is defined by mass, drive power and maximum speed : no consideration is given to design details, the analysis assumes a tractive force simply defined by the traction power. These details derive the mean climber speed on each tether segment and hence the time to ascend each segment. Spreadsheet logic then allocates multiple climber masses on elements at variable travel time spacings, for example 24 hour spacings for climbers despatched from the Earth Port once per day. The effective weight of each climber yields an additional tether tension force, giving the total tension in the tether at any altitude. Spreadsheet enhancements include an algorithm for daylight duration at varying altitudes, variable with the time of year : this permits an option for climber spacing to be calculated for solar-powered climbing. The impact of input parameters (climber mass, power and maximum speed, departure intervals, continuous or daylight-only climbing, etc.) yield outputs such as maximum tether tension and climb time. The value of this technique becomes apparent when inputs are adjusted to yield similar tether tensions, representing a scenario with a maximum tether stress limit. It is possible to find, for example, how the maximum climber gross mass varies with maximum speed or drive power. Examples of findings include (1) the benefits of higher power and maximum speed are complex, and highly dependent on the climber power/weight ratio (2) 24-hour climbing might allow 20% more payload (for any given tether strength and climber design) compared with daylight-only climbing (3) two smaller climbers launched daily might enable 15% more payload to be raised compared with a single daily launch. Such deeper understanding of climber dynamics highlights the complexity of climber design optimisation : key parameters are the net climber power/mass ratio and the maximum climber speed.

Keywords: Space Elevator, Climber, Payload, Dynamics

1. INTRODUCTION

The primary function of any Space Elevator (SE) tether is to support the weight of climbers and enable motion of those climbers. Previous studies and papers [1] [2] have shown that the peak tether stress is optimised by the tether having a variable cross-sectional area defined by a 'taper equation' : a tether with such a profile can have constant stress from the Earth's surface ('Earth Port') to the counterweight ('Apex Anchor').

The taper equation used in many previous studies yields a constant ('equilibrium') tether stress at all altitudes, usually selected to be the maximum working stress of the tether material. The tether total mass depends on the chosen tensile force at the Earth Port, sufficient to support the weight of a climber plus a margin for tether retention. This design strategy is good for a single climber, but the weight of extra climbers at higher altitudes results in an increase in the tether stress : Figure 1 shows an example of a tether scaled for an equilibrium stress of 88 GPa with an Earth Port force of 35 tonne-f. The solid line shows the stress with one climber, other lines show the stress with six distributed climbers for the same and lower retention forces.

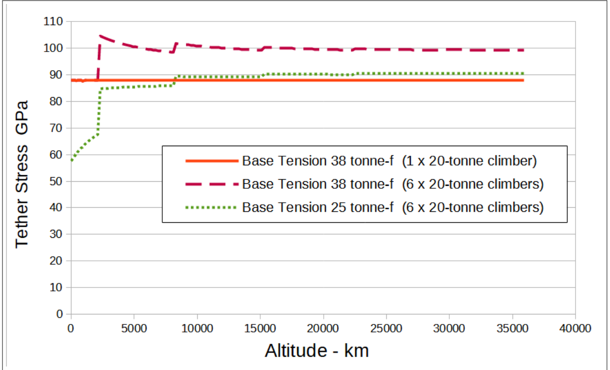


Figure 1 : Tether Stress with 1 or 6 climbers on arbitrary tapered tether

It can be seen that the positions of multiple climbers on the tether determines the peak tether stress and hence the required design of the tether itself. The present SE concept [2] has daily departures of climbers from Earth, climbing to GEO of 7 days : the number and altitude of the climbers will depend on many variables, including climber power, mass and maximum speed.

This paper describes work using a spreadsheet tool to study the effect of climber performance parameters at steady state conditions and draw conclusions on the optimisation of climber design and operational details. The spreadsheet itself will not be published, but sufficient detail is given to permit it to be reproduced.

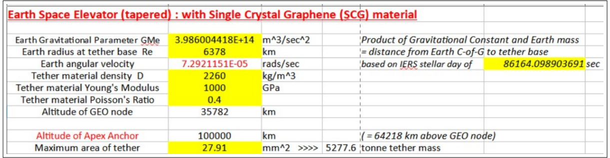
An analytic (equation-based) paper was published by Shelef in 2012 [3] with similar conclusions.

2. METHODOLOGY

The method used a spreadsheet-based finite-element-type analysis, with the tether divided into segments of variable length. This first calculated the element cross-sectional areas and stresses, then applied additional loading representing the weight of climber.

2.1 Tether Element Areas, Masses and Tension

Spreadsheet input data allowed tether element gravity and centrifugal forces to be calculated as in earlier studies, as shown in Figure 2 below.

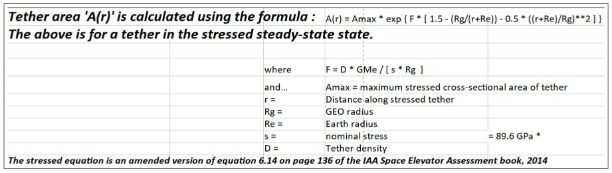


Earth Space Elevator (tapered) : with Single Crystal Graphene (SCG) material

Earth Gravitational Parameter GMe	3.986004418E+14	m^3/sec^2		*Product of Gravitational Constant and Earth mass*		
Earth radius at tether base Re	6378	km		*= distance from Earth C-of-G to tether base*		
Earth angular velocity	7.2921151E-05	rads/sec		*based on IERS stellar day of*	*86164.098903691*	*sec*
Tether material density D	2260	kg/m^3				
Tether material Young's Modulus	1000	GPa				
Tether material Poisson's Ratio	0.4					
Altitude of GEO node	35782	km				
Altitude of Apex Anchor	100000	km		*(= 64218 km above GEO node)*		
Maximum area of tether	27.91	mm^2 >>>>	5277.6	tonne tether mass		

Figure 2 : Input data for gravity & centrifugal forces

The tether taper equation in the spreadsheet used additional input data for the tether material : predicted properties for graphene super-laminate (formerly known as single crystal graphene) were used, see above.



Tether area 'A(r)' is calculated using the formula : A(r) = Amax * exp { F * [1.5 - (Rg/(r+Re)) - 0.5 * ((r+Re)/Rg)**2] }

The above is for a tether in the stressed steady-state state.

where F = D * GMe / [s * Rg]

and... Amax = maximum stressed cross-sectional area of tether

r = Distance along stressed tether

Rg = GEO radius

Re = Earth radius

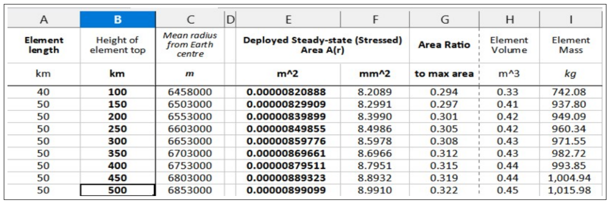
s = nominal stress = 89.6 GPa *

D = Tether density

The stressed equation is an amended version of equation 6.14 on page 136 of the IAA Space Elevator Assessment book, 2014

Figure 3 : Taper Equation and other input data

The above was used to build the basic model of the tether. For each element the length was specified, then added to the altitude of the previous (lower) element to yield a mean radius from the Earth centre : the taper equation gave the element area and volume, yielding the mass. Figure 4 below shows the layout of this section of the spreadsheet at a region of the tether from the top of the atmosphere to 500 km altitude.

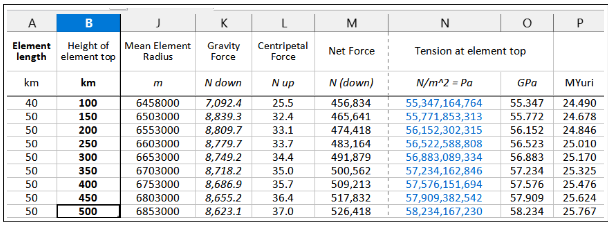


A	B	C	D	E	F	G	H	I
Element length	Height of element top	*Mean radius from Earth centre*		**Deployed Steady-state (Stressed) Area A(r)**		**Area Ratio**	Element Volume	Element Mass
km	**km**	*m*		**m^2**	**mm^2**	**to max area**	m^3	*kg*
40	**100**	6458000		**0.00000820888**	8.2089	0.294	0.33	742.08
50	**150**	6503000		**0.00000829909**	8.2991	0.297	0.41	937.80
50	**200**	6553000		**0.00000839899**	8.3990	0.301	0.42	949.09
50	**250**	6603000		**0.00000849855**	8.4986	0.305	0.42	960.34
50	**300**	6653000		**0.00000859776**	8.5978	0.308	0.43	971.55
50	**350**	6703000		**0.00000869661**	8.6966	0.312	0.43	982.72
50	**400**	6753000		**0.00000879511**	8.7951	0.315	0.44	993.85
50	**450**	6803000		**0.00000889323**	8.8932	0.319	0.44	1,004.94
50	**500**	6853000		**0.00000899099**	8.9910	0.322	0.45	1,015.98

Figure 4 : Tether Calculations (1)

The gravity and centrifugal forces on each element were calculated and used to derive the element stress, based on the mean element radius, mass, Earth gravity and rotation (as Figure 2) and the tension force from the next lowest element. The tension force in the lowest element was usually a manual entry : a value of 260 kN plus the weight of a 20t climber at 60km was used for many of the example calculations in this paper.

Figure 5 shows the results of these calculations : the Tension stress (Pa) is the Net Force (N) divided by the Area (m^2, see Figure 4), with a conversion to GPa and specific stress (MYuri) included for convenience.



A	B	J	K	L	M	N	O	P
Element length	Height of element top	Mean Element Radius	Gravity Force	Centripetal Force	Net Force	Tension at element top		
km	**km**	*m*	*N down*	*N up*	*N (down)*	*N/m^2 = Pa*	*GPa*	MYuri
40	**100**	6458000	*7,092.4*	25.5	456,834	55,347,164,764	55.347	24.490
50	**150**	6503000	*8,839.3*	32.4	465,641	55,771,853,313	55.772	24.678
50	**200**	6553000	*8,809.7*	33.1	474,418	56,152,302,315	56.152	24.846
50	**250**	6603000	*8,779.7*	33.7	483,164	56,522,588,808	56.523	25.010
50	**300**	6653000	*8,749.2*	34.4	491,879	56,883,089,334	56.883	25.170
50	**350**	6703000	*8,718.2*	35.0	500,562	57,234,162,846	57.234	25.325
50	**400**	6753000	*8,686.9*	35.7	509,213	57,576,151,694	57.576	25.476
50	**450**	6803000	*8,655.2*	36.4	517,832	57,909,382,542	57.909	25.624
50	**500**	6853000	*8,623.1*	37.0	526,418	58,234,167,230	58.234	25.767

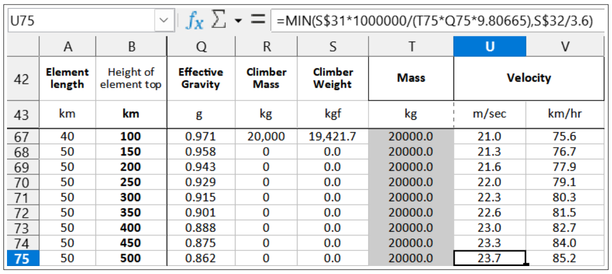
Figure 5 : Tether Element Tension Calculations

2.2 Climber Motion and Power

The spreadsheet then included calculations that are perhaps more novel to this analysis. Figures 6 first shows an 'effective gravity' calculated for reference, based on the difference between gravity and centrifugal forces expressed as a proportion of standard sea-level gravity. Mass (kg) and weight (kgf) values are shown in columns R and S, representing a climber assumed to be supported by the tether at the start of the climb and every 24 hours thereafter, determined using logic discussed below.

Column T in Figure 6 includes the climber mass at every element altitude, used in a calculation of climber speed shown in columns U and V based on the fundamental relationship 'Power = Force x Velocity = Weight x Climb Speed', amended by a logic function shown in Figure 6 to not exceed a specified maximum climb speed (see Figure 7 and discussion below).

Note, values for mass in column T are in every row to permit manual mass changes during the climb, perhaps due to components being jettisoned, but no such changes were used in the calculations in this paper.



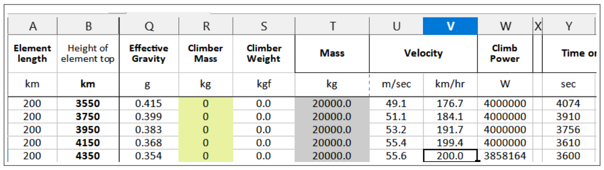
U75 =MIN(S$31*1000000/(T75*Q75*9.80665),S$32/3.6)

	A	B	Q	R	S	T	U	V
42	**Element length**	Height of element top	**Effective Gravity**	**Climber Mass**	**Climber Weight**	**Mass**	**Velocity**	
43	km	**km**	g	kg	kgf	kg	m/sec	km/hr
67	40	**100**	0.971	20,000	19,421.7	20000.0	21.0	75.6
68	50	**150**	0.958	0	0.0	20000.0	21.3	76.7
69	50	**200**	0.943	0	0.0	20000.0	21.6	77.9
70	50	**250**	0.929	0	0.0	20000.0	22.0	79.1
71	50	**300**	0.915	0	0.0	20000.0	22.3	80.3
72	50	**350**	0.901	0	0.0	20000.0	22.6	81.5
73	50	**400**	0.888	0	0.0	20000.0	23.0	82.7
74	50	**450**	0.875	0	0.0	20000.0	23.3	84.0
75	50	**500**	0.862	0	0.0	20000.0	23.7	85.2

Figure 6 : Climber Weight and Velocity

Figure 7 is an extended version of Figure 6 at a higher altitude, showing in column W the climb power derived from the climb speed. Column Y shows the time taken for the climber to ascend each tether element, simply derived from the mean velocity and element length.

Note that at 4350km altitude the climb velocity has reached the specified maximum of 200 kph, resulting in the required climb power being reduced from the 4000 kW maximum to a value derived from the velocity (200 kph) and the climber weight (mass x effective gravity).



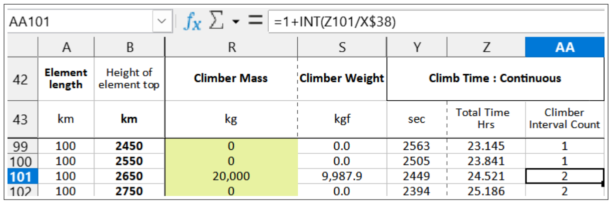
A	B	Q	R	S	T	U	V	W	X	Y
Element length	Height of element top	**Effective Gravity**	**Climber Mass**	**Climber Weight**	**Mass**	**Velocity**		**Climb Power**		**Time o**
km	**km**	g	kg	kgf	kg	m/sec	km/hr	W		sec
200	**3550**	0.415	0	0.0	20000.0	49.1	176.7	4000000		4074
200	**3750**	0.399	0	0.0	20000.0	51.1	184.1	4000000		3910
200	**3950**	0.383	0	0.0	20000.0	53.2	191.7	4000000		3756
200	**4150**	0.368	0	0.0	20000.0	55.4	199.4	4000000		3610
200	**4350**	0.354	0	0.0	20000.0	55.6	200.0	3858164		3600

Figure 7 : Required power and element ascent time

The climber transit times for each element can be summed to derive total ascent times to GEO. For continuous climbing (with some power source not dependent on direct solar energy) this is simply the total of column Y as shown on Figure 7 above.

2.3 Climber Placement : Continuous Climbing

For climbers departing the Earth Port in N second time intervals the mass of an additional climber must be added to column R (Figure 6) on rows when the total of column Y exceeds integer multiples of N. This is achieved using a simple logic formula shown in Figure 8 below :



AA101 =1+INT(Z101/X$38)

	A	B	R	S	Y	Z	AA
42	**Element length**	Height of element top	**Climber Mass**	**Climber Weight**	**Climb Time : Continuous**		
43	km	**km**	kg	kgf	sec	Total Time Hrs	Climber Interval Count
99	100	**2450**	0	0.0	2563	23.145	1
100	100	**2550**	0	0.0	2505	23.841	1
101	100	**2650**	20,000	9,987.9	2449	24.521	2
102	100	**2750**	0	0.0	2394	25.186	2

Figure 8 : Weight location logic. Columns T-X hidden.

In the above it can be seen that the element climb time is summed and converted to hours in column Z. The formula given for cell AA101 shows how the Climber Count steps from 1 to 2 when the climb time exceeds the value set in cell X38 (manual entry, not shown).

The climber mass in cell R101 is equal to the climber mass in column T (Figure 6) multiplied by the difference between cells AA101 and AA100, and so is only non-zero when the climb time to that altitude increases by a multiple of the climber departure interval.

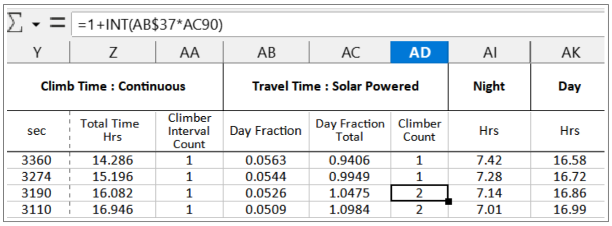
The climber weight in column S is the mass multiplied by the effective gravity (column Q, Figure 6) : this is then used to increase the tether tension force at that altitude (column M, Figure 5).

2.4 Climber Placement : Solar-Powered Climbing

Section 2.3 described how multiple climber weights are automatically added to the tether tension for continuously-powered climbing, but non-stop ascent is not the only operational concept : climbing powered by direct solar power is the preference of many researchers, requiring additions to the spreadsheet. These additions provide an option for alternate climber positions representing 'over-night' stops when the climber is shielded from the Sun by the Earth, resulting in closer climber spacing for any given Earth Port departure interval.

These additional calculations are based on the algorithm for 'night-time' duration devised by Dr John Knapman as described in his 2013 ISEC Paper [4]. The calculation inputs include the Earth diameter, climber altitude and solar declination : the latter depends on the time of year, varying from zero at the equinox to a maximum of 23.5° at the solstices. The equations are only valid for locations vertically above the equator.

Figure 9 below shows the section of the spreadsheet dealing with day/night durations and associated logic. Columns Y, Z and AA are as previously described : column AI contains a night duration time derived using the Knapman algorithm, with day time then simply calculated in column AK by subtraction from 24 hours. The element climb time in column Y is then divided by the Day duration to yield a 'Day Fraction' value in AB, with a running total in AC. This value is approximately the fraction of local day length, and can be used to determine the altitude reached daily with solar-powered climbing by means of the 'Climber Count' calculation in column AD.



=1+INT(AB$37*AC90)

Y	Z	AA	AB	AC	AD	AI	AK
Climb Time : Continuous			**Travel Time : Solar Powered**			**Night**	**Day**
sec	Total Time Hrs	Climber Interval Count	Day Fraction	Day Fraction Total	Climber Count	Hrs	Hrs
3360	14.286	1	0.0563	0.9406	1	7.42	16.58
3274	15.196	1	0.0544	0.9949	1	7.28	16.72
3190	16.082	1	0.0526	1.0475	2	7.14	16.86
3110	16.946	1	0.0509	1.0984	2	7.01	16.99

Figure 9 : Day/Night Calculation & Logic

As in Section 2.3, a change in the integer value of the value in column AD represents the altitude at which a non-zero climber mass is required in column R.

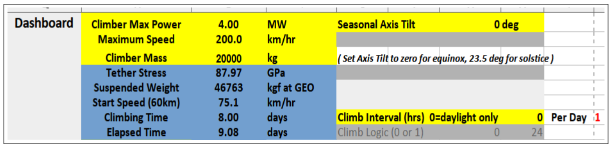
Also as before, more than one solar-powered departure from the Earth Port each day can be modelled by multiplying the Day Fraction total by the number of daily departures, leading to more frequent stepping of the Climber Count integer.

Note that this 'Day Fraction' methodology is not exact, especially close to the Earth where daylight duration is changing rapidly with altitude, but it does yield an approximate result that is consistent between comparisons of different design inputs. Greater precision could be achieved by reducing tether element height, leading to more rows in the spreadsheet and hence a larger file size.

2.5 Input and Output Dashboard

The calculations in preceding sections describe how the velocity and weights of multiple climbers can be found for all altitudes on the tether, yielding values of total tether stress and climb times. Note, work to date has only included motion from the Earth Port to GEO : climber dynamics from GEO to the Apex Anchor will require different methodology.

To assist in analysis of the data a 'Dashboard' section of the Spreadsheet was created, as shown in Figure 10 below. Fields in Yellow were for data inputs and can be seen to include climber maximum traction power (the rate of work done by the climber on the tether), the maximum climber speed (a practical or operational climber design limitation) and the climber gross mass. Other fields are logic inputs for specifying details of solar powered or continuous climbing, with intermediate calculations hidden in Grey cells.



Dashboard	**Climber Max Power**	**4.00**	**MW**	**Seasonal Axis Tilt**	**0 deg**			
	Maximum Speed	**200.0**	**km/hr**					
	Climber Mass	**20000**	**kg**	*(Set Axis Tilt to zero for equinox, 23.5 deg for solstice)*				
	Tether Stress	**87.97**	**GPa**					
	Suspended Weight	**46763**	**kgf at GEO**					
	Start Speed (60km)	**75.1**	**km/hr**					
	Climbing Time	**8.00**	**days**	**Climb Interval (hrs) 0=daylight only**		**0**	**Per Day**	**1**
	Elapsed Time	**9.08**	**days**	Climb Logic (0 or 1)	0	24		

Figure 10 : Input and Output Dashboard

Fields highlighted in Blue are output parameters : the Tether Stress is the maximum value from column O, usually found at GEO (as the tether stress derives from the weight of both the climbers and the tether itself), although Figure 1 shows that the peak stress could occur at a lower altitude.

The 'Suspended Weight' figure is the sum of the climber weights from column S, of interest as it does not include the weight of the tether itself. The 'Start Speed' is the steady-state velocity of the climber on the lowest element (after the initial acceleration from rest), based on the climber power and mass. The 'Climbing Time' is the time spent climbing : the 'Elapsed Time' is the same as the climbing time if climbing is continuous, but is greater if climbing stops during 'Night' periods.

The Dashboard allowed rapid post-processing of the spreadsheet results : differing inputs could readily be input and the results of parameter changes immediately viewed and copied for further analysis.

The Dashboard does not include any tether design parameters such as taper ratio, maximum cross-sectional area, material properties or Earth Port tension : these can readily be changed elsewhere, but the primary focus of the spreadsheet at this stage is on the climber design.

3. ANALYSIS

Analysis was undertaken using manual inputs into the Dashboard, with outputs then copied to a separate sheet for further processing and interpretation.

3.1 *Effect of Independent Variables*

Three independent climber design variables (climber power, maximum speed and mass) were adjusted separately, with results shown in Figures 11-13 below.

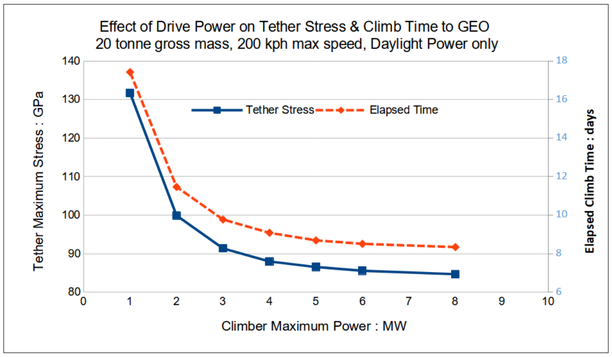


Figure 11 : Effect of Tractive Power

Low tractive powers can be seen to lead to very high tether stresses and climb times : lower powers mean slower climbing at low altitudes, hence more climbers at low altitudes and more suspended weight. Conversely, greater tractive powers have a diminishing effect on both climb time and stress : the limiting maximum speed is reached earlier as power increases, reducing the benefit of the higher available powers.

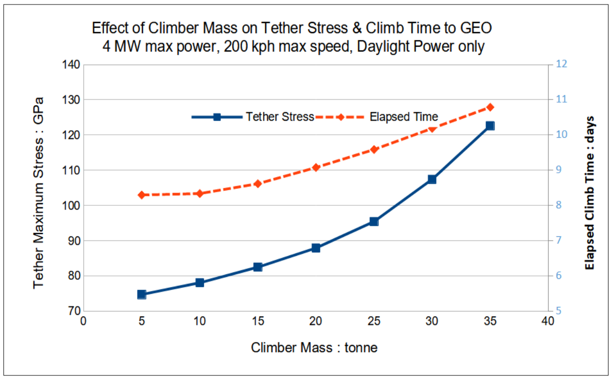


Figure 12 : Effect of Climber Mass on Tether Stress and Climb Time

Figure 12 shows that reducing Climber Mass for the same power steadily reduces the tether tension as the suspended weight falls, though the stress cannot reduce below that required to support the tether weight alone. Higher climber masses lead to greater tether stresses and climb times as the initial climb speed falls and more climbers are suspended at lower altitudes.

Figure 13 shows the effect of varying the maximum climber speed : this speed is assumed to be independent of the climber mass, and a limit imposed by design features such as wheel stresses (for a wheeled climber), steering limitations or other control system factors.

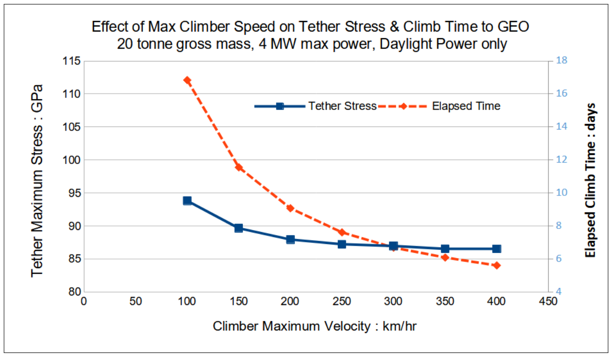


Figure 13 : Effect of Maximum Climber Speed

Increasing the maximum climb speed can be seen to have little impact on the tether stress as the majority of the suspended climber weight is made up from the climbers on the lower parts of the tether before the higher climb speeds are achieved. The maximum speed has a far greater impact on the total time for climbing to GEO : this is perhaps best explained by considering Figure 14 below, showing the climber altitude plotted against climb time for a number of maximum speeds.

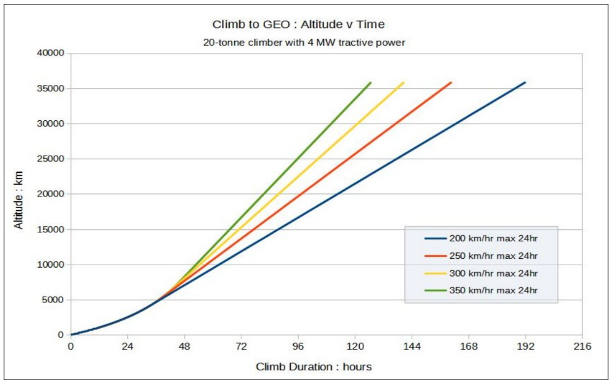


Figure 14 : Climber Altitude v Time, 200-350 kph

This plot clearly shows that increasing the climber maximum speed (for a 20 t, 4 MW climber) has no significant effect on climber altitude until almost the end of the 2nd day of continuous climbing, at an altitude over 5000 km. At this altitude the effective gravity has fallen to less that 30% of that at the surface, explaining the small impact of maximum speed on tether stress.

3.2 Example of Multi-Variable Analysis

The Dashboard permits more complex analysis of the impact of input variables without post-processing. Figure 15 below is an example of such work : the data for this plot was collected by manual adjustment of the climber maximum speed to yield specific values of climb time to GEO, with this repeated for a range of climber power values. This data led to this plot of the maximum speed required for a range of target climb times, shown against specific power.

It can be simply be shown that climb velocity, and hence climb time, is a function of the specific power (maximum climber tractive power / mass) at speeds below the maximum speed. Figure 15 shows plots of climbing time to GEO against the two independent variables of maximum climb speed and specific power.

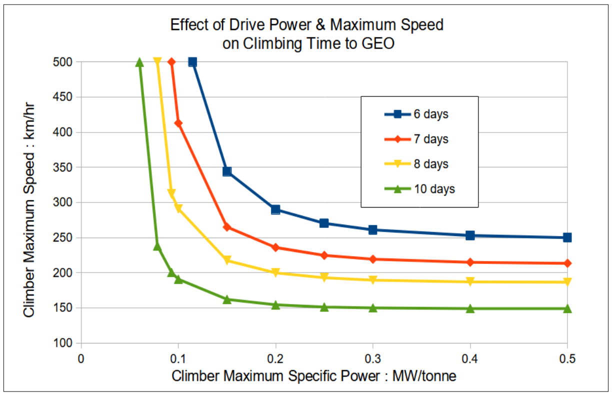


Figure 15 : Max Speed v Specific Power, constant ascent time

The conclusions from this plot are similar to those from Figures 11 and 13 :

- low climber powers (below 0.1 - 0.15 MW/t) will result in high ascent times to GEO with increasing maximum speed have little effect.
- high climber powers (above 0.25 – 0.3 MW/t) will not reduce climb times to GEO, these times becoming almost solely dependent on the maximum climb speed.

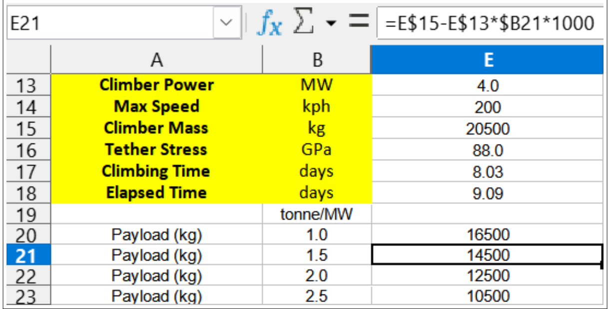
The results presented on the preceding Figures show the effect of varying the 3 independent input variables, and it is clear that climb time, tether stress and other responses have a highly non-linear response to each.

These simple analyses do not permit any meaningful climber design optimisation : what is needed is a means of studying and optimising climber design parameters with constant values of tether stress. Climber payload capability has also not yet been addressed.

3.3 Responses with Constant Tether Stress

3.3.1 Post-Processing : Climber Payload

More detailed understanding of the tether and climber dynamics can be achieved by simple post-processing of data harvested from the Dashboard. One important parameter is the Climber Payload, simply defined as the difference between the net and gross climber masses. The net climber mass is determined by the climber power and the climber 'specific mass'. The payload is calculated as shown in Figure 16 below.


E21 =E$15-E$13*$B21*1000

	A	B	E
13	**Climber Power**	MW	4.0
14	**Max Speed**	kph	200
15	**Climber Mass**	kg	20500
16	**Tether Stress**	GPa	88.0
17	**Climbing Time**	days	8.03
18	**Elapsed Time**	days	9.09
19		tonne/MW	
20	Payload (kg)	1.0	16500
21	Payload (kg)	1.5	14500
22	Payload (kg)	2.0	12500
23	Payload (kg)	2.5	10500

Figure 16 : Data Post-Processing for Climber Payload

In the above figure the data in cells E13-E18 were copied from the Dashboard. The daily Payload was then calculated using the net climber specific mass (tonne/MW) in cells B20 and below : multiplication of the specific mass by the climber power yields the net mass of the (unladen) climber, subtraction of this from the gross climber mass yields the 'Payload' mass as shown in cells E20 and below.

The relationship between Payload and Specific Mass is linear, as shown in Figure 17 below.

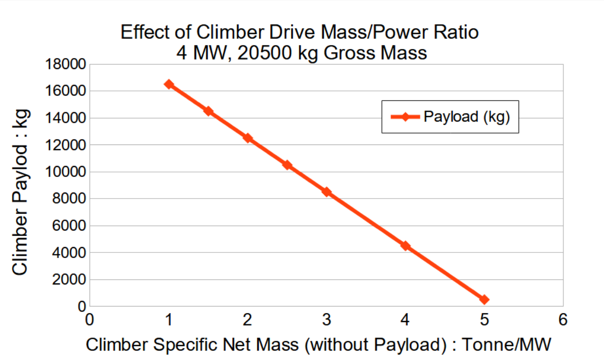


Figure 17 : Payload .v. Specific Net Mass

This shows the importance of the 'Specific Mass', highlighting the need to minimise mass in any climber design : in this example 4 MW daily climbers with a specific mass of 5.0 tonne/MW would be able to carry only 500kg payload at the target tether stress.

3.3.2 Climber Payload Optimisation

The previous section highlighted the need for climber mass reduction, but other climber design and operational concepts should also be considered to maximise the payload : for example, the gross climber mass need not be fixed in a Space Elevator operational scenario, as in practice the tether stress will be the limiting factor, with system mass-raising capability maximised with the tether loaded to that limit.

The automated climber-positioning feature of the spreadsheet allows for input parameters to be manually adjusted to yield the target tether stress, with a value of 88 GPa chosen : for the purpose of this paper the absolute value of the chosen stress is immaterial as the tether properties do not impact the climber dynamics.

The results shown in Figure 18 below were extracted from the dashboard as described in section 3.3.1 : the maximum climb speed and climber power were set manually, then the gross climber mass adjusted to yield the target tether stress. The data sets were then copied to a separate worksheet for payload calculation and chart plotting.

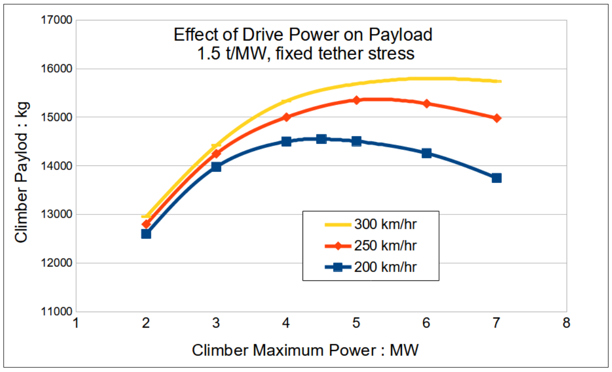


Figure 18 : Payload Trends .v. Power & Max Speed

It can be seen that for each climber maximum speed there is an optimum power for maximum payload : at lower powers the climbers will be closer together in the early stages of the ascent and so have a higher total weight, whereas at higher powers the extra mass will have less benefit as the maximum speed is reached earlier. This last reason explains why a climber with a higher maximum speed will benefit more from a higher power during the early days of the ascent.

Figure 18 was generated for a specific mass of 1.5 tonne/MW, chosen as it corresponds to the current ISEC baseline concept of a 4 MW climber with a 6 tonne unladen mass [4] [5]. Figure 19 shows the results from post-processing for a range of specific masses.

Each line plotted on Figure 19 was generated using the same spreadsheet output data, with the different specific mass figures explored in the post-processing worksheet. (Note also that the data points for 4 MW maximum power on Figure 19 are the same as those plotted earlier on Figure 17.)

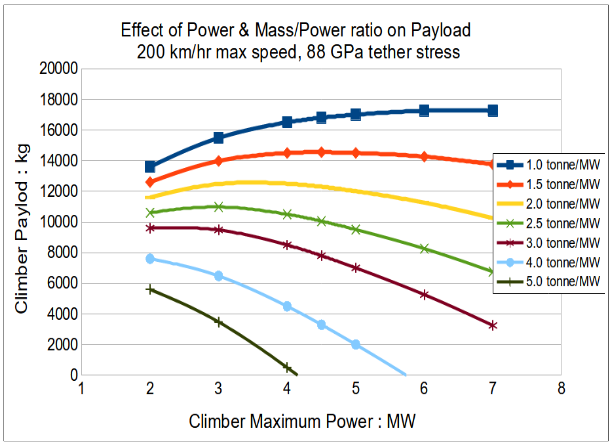


Figure 19 : Effect of Specific Mass on Payload

The observation in section 3.3.1 that a 4MW climber at 5 tonne/MW could carry little payload can be seen now to be an over-simplification : a climber with that specific power rated at only 2 MW would be able to carry a payload of 5600 kg for the same tether stress, with a total gross mass of 15600 kg.

At the other extreme it can be seen that climbers with a low specific mass (tonne/MW) will not only carry more payload (for the same tether stress and total power), but will also be more effective if they are more powerful : in practice this will mean that the initial climb speed is higher, meaning greater vertical spacing between the daily climbers and so fewer climbers that need to be supported by the tether.

This reiterates the conclusion that climber specific mass (tonne/MW) is a key design parameter. Lower specific mass will increase the daily payload to GEO, with optimum performance at higher powers.

It is inevitable that early Space Elevator climber designs and technologies will be less capable than those that are developed in later years. It therefore follows that the lift capacity of any fixed space elevator tether will improve over time as climber systems, materials and power supply technologies reduce the climber specific mass.

3.3.3 Effect of Time of Year on Payload

The results presented in the previous sections have all been for solar-powered climbers departing the Earth Port at the Equinox : this is the worse-case time of year as The Sun is eclipsed by The Earth at all altitudes.

Figure 20 below presents a plot of potential payload of a solar-powered 250 kph, 1.5 t/MW climber (as previously plotted in Figure 18) compared with the payload possible with the same climber at the solar solstices. This new data set was obtained by adjusting the 'Seasonal Axis Tilt' spreadsheet input (see Figure 10) and post-processing for payload as before.

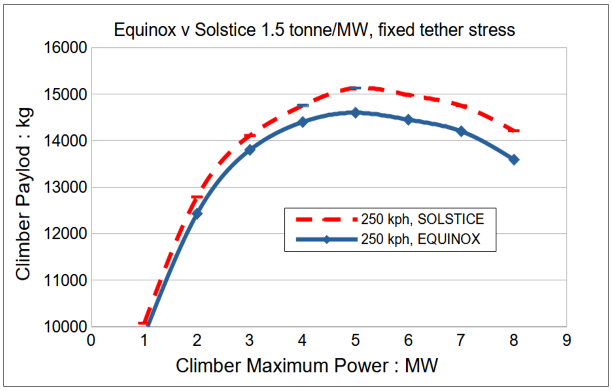


Figure 20 : Payload Capacity with Time of Year

This chart shows a potential payload improvement of the order of 500 kg, increasing slightly at higher climber tractive powers. This can be understood by considering Figure 21 below, showing the hours of local 'Night' for each climber after each day of ascent.

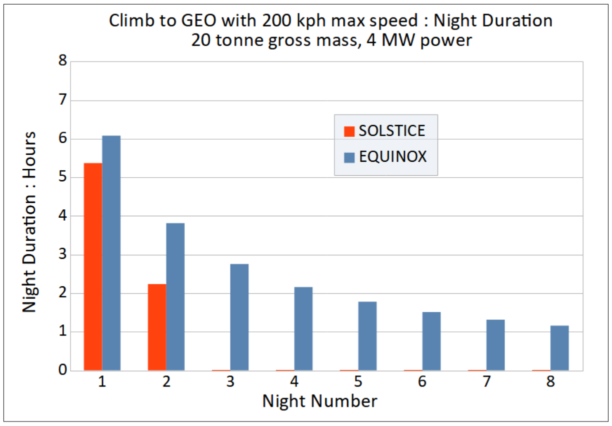


Figure 21 : Night Duration after each day of ascent

It can be seen that after the third day of ascent the climber will be in permanent sunlight at the solstice : this means the weight of the higher climbers will be slightly reduced, lowering the tether stress or allowing a greater payload to be carried.

3.3.4 Continuous .v. Solar Powered Climbing

The 'Time of Year' effect described in the previous section is not a design parameter, so is included mainly as an observation. A far greater impact on potential climber payload, and one that is definitely influenced by design choices, is a switch from solar-powered climbing with overnight stops to continuous climbing without any overnight stops.

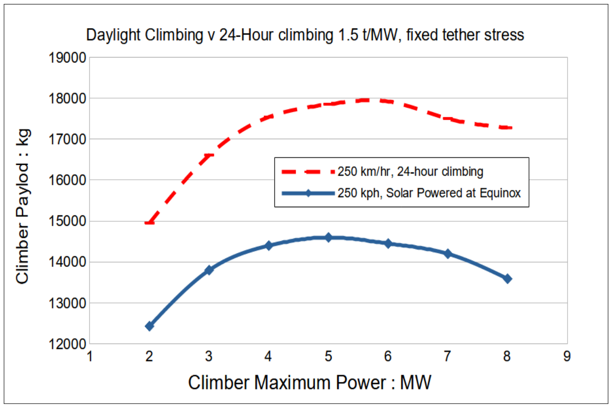


Figure 22 : Solar Powered .v. Continuous Climbing

As with previous plots, Figure 22 presents results with the gross climber mass adjusted to yield the same maximum tether stress and the payload calculated using post-processing. The continuous '24-hour climbing' payload is of course independent of the time of year : comparison with Figure 20 shows that the benefit over solar-powered climbing would be less at the solstices, but still a very substantial 18% payload increase (14755 to 17540 kg at 4MW maximum power).

The magnitude of the payload benefit of continuous climbing is dependent on other parameters (maximum speed and specific net mass), the reason becoming clear when the climber weights are considered. The lowest climber will always be subject to 1g, but the next climber up the tether (that departed 24 hrs earlier) will be subject to either 0.634g at the 1650 km altitude reached before local sunset or 0.5g at 2650 km reached after 24 hours of continuous ascent. The effective gravity for the next climber up similarly falls from 0.34g to 0.22g, again contributing less to the weight on the tether and the corresponding peak stress.

The means of achieving continuous climbing is outside the scope of this report : new technologies may allow some form of onboard power source to become feasible, or power could be transmitted to the climber from either the Earth or space. The optimum solution may even be hybrid, with transmitted power at low altitudes and solar power at high altitudes (at which less power would be required).

3.3.5 Climber Earth Departure Frequency

All previous examples have been for a single daily climber departure leaving the Earth at 24 hours intervals at local dawn when solar-powered. Tether stresses are only calculated at the departure time as tether loading will reduce during the day as climbers ascend and are subject to lower effective gravity. This corresponds to the default system operating concept as described in earlier work, but it is not the only option.

Figure 23 shows a plot of payload, again calculated using the post-processing method described earlier, for increasing climber launch frequencies equally spaced through daylight hours or a 24-hour period. In each case the total climber maximum power per day is held constant at 4 MW : this means that for N launches per day the power of each climber was set to 4/N MW. Plots are shown for solar-powered day-time climbing with 200 and 250 kph maximum speed, and for continuous climbing at 250 kph maximum.

Note that the 250kph data for 1 climb/day is the same as the 4MW data in Figure 22. Note also that the payload calculated for the 200kph at 1 climb/day is exactly 14000 kg : this is as the tether strength and stress limit were chosen to yield the ISEC operational concept of a single daily solar-powered 20t climber with 14t payload with 4MW maximum power (= 1.5 t/MW).

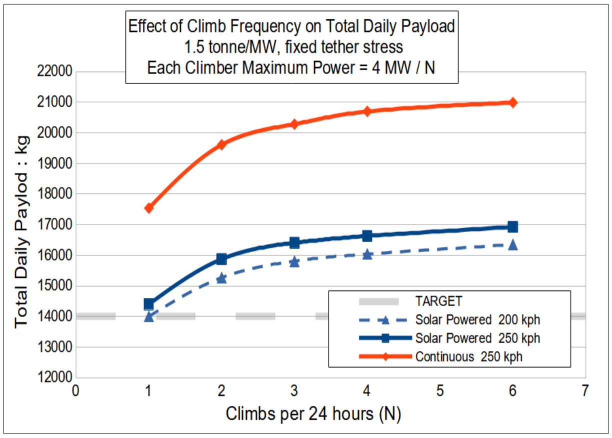


Figure 23 : Climber Frequency .v. Total Daily Payload

The conclusion from Figure 23 is that having multiple daily launches, preferably with a continuous power supply, would maximise the daily payload mass raised to GEO. This supports the design concept of the climber consisting of drive modules : these could be used as small individual climbers whenever the cargo can be split into small separate payloads, but could be connected together to make a larger climber assembly whenever the cargo cannot be subdivided.

A change of climber concept from solar-powered to continuous power would inevitable alter the specific net mass of the climber. The mass of some components may not change significantly (such as wheels, motors, cooling systems, etc), but the mass of solar arrays and perhaps associated power systems would alter. The change would depend on the chosen power source, for example power transmitted via the tether could well result in a lower climber mass. Receivers for beamed power may have a mass less or similar to that of solar arrays, whereas on-board power sources may well require significantly more mass : further discussion on power source options is outside the scope of this paper.

Figure 24 below presents the same '250 kph and 1.5t/MW' data sets as Figure 23, but now includes a dataset for a higher specific net mass. The figure of 2.28 t/MW was chosen to yield the same daily payload as the solar-powered variant for a single daily climber.

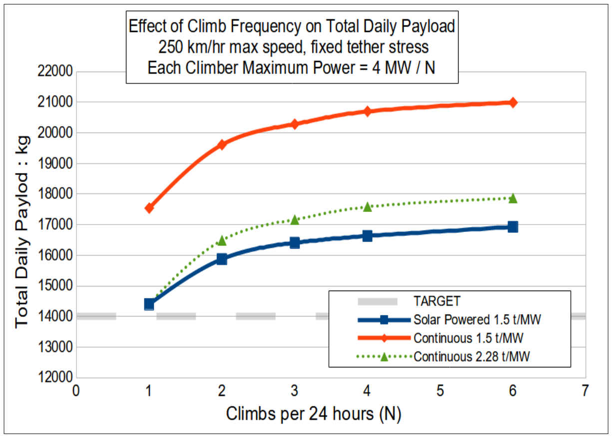


Figure 24 : Continuous Climbing, higher specific mass

The 2.28 t/MW specific net mass corresponds to a 4 MW climber with a net mass of 9120 kg, compared with the baseline mass of 6000 kg. It can be seen that the benefits of multiple climbs per day are greater with continuous climbing.

The optimum number of climber departures per day will depend on many factors, including climber design and operational requirements. If the climber drive modules are small then more climbs per day would be feasible, and smaller modules may lead to economies of scale in manufacturing and lower development costs, etc, but operational requirements may dictate fewer departures per day if cargo cannot readily be packaged into smaller payload units.

3.3.6 Other Functionality Options

During the course of spreadsheet development other functionality was added to address specific climber design concepts such as a motor/wheel friction drives or linear induction motors (LIM) systems The results of this work is summarised as follows :

- The dashboard was extended to include details of electric motor number, power, maximum torque and speed characteristics. An algorithm calculated the corresponding wheel radius based on the climber weight at 1g and torque: the tractive power was then derived from the motor curve. The motor useful speed range then set the maximum climber speed.
  This change meant that the climber max speed, payload and climb time became dependent on the climber mass and motor characteristics : these dependencies are complex and are not presented here, but the main conclusion was that motor speed range was another key parameter and must be a priority in motor selection.

- The limited speed range of some available electric motors led to the concept of some drive wheels being of larger diameter, allowing a higher maximum climb speed at high altitudes with the smaller wheels disengaged. Logic was added to model this concept, but benefits were found to be minimal.

- Consideration of LIM drives [5] concluded that they could not easily be used on a light-weight tether near the Earth, at least not with any significant useful power, but they potentially could permit a higher maximum speed. The option was added to the spreadsheet to switch the maximum climb speed to a higher value corresponding to a LIM drive, enabled by logic based on the required climb power.
  A typical model using this feature included a 1.0 MW LIM drive with a 400-500 kph speed limit alongside a 4.0 MW main friction drive with a 200-250 kph speed limit. The transition to LIM-only power was found typically to be well above 10,000 km altitude and not reached until the 3rd or 4th day of climbing : the payload benefit of the additional LIM drive speed was small and likely to be offset by the presumed extra LIM system mass. This analysis led to the conclusion that a friction/LIM hybrid system did not warrant more detailed study, so is not described in any more detail here.

4. DISCUSSION

4.1 Limitations of Analysis Methodology

Limitations of the numeric spreadsheet analysis technique described in this paper include accuracy and lack of dynamic effects. As with any finite element method the accuracy depends on the size of elements used, or in this case the length of each tether segment.

The spreadsheet version used for much of the reported analysis comprised some 200 elements of variable length between the Earth and GEO (35,900 km) : these element lengths were smaller close to the Earth as weights and other parameters were varying rapidly there, with lengths typically increased to 100 km at 1000 km and 200 km at 3000 km.

Many calculations, for example for climber weight, were based on the conditions at the mid-point of each element, and these approximations will have a small impact on the calculated tether stresses. A more significant potential error is that associated with climber positioning : the logic used to add a climber weight to any specific element was based on the climb time passing a threshold value, so a small change in an input parament could lead to the climber position stepping to an adjacent element with a corresponding step change in peak tether stress.

The climber positioning errors were more significant when multiple climbs per day were assessed: for example, with 6 climbs per day (4 hour spacing) the climbers could be only 3 or 4 elements apart.

These errors would have been minimised by rebuilding the spreadsheet with far more elements, improving accuracy. However, the primary value of the work performed to date has been to compare alternative design and operational options, and these overview comparisons are believed to be valid.

4.2 Alternative Methodologies

The tether stress profile in the tether has been found using precise equation-based analytic methods by many researchers, but for a tether loaded only by its own weight and a fixed tension at the lower end. Analytic equations have also been devised to yield climber motion along the tether, but only for a fixed power.

These two analytic methods could be combined to yield results similar to the element-based spreadsheet technique, perhaps by building them into customised modules within some commercial 1-D dynamic simulation software. Many such software packages exist with varying degrees of user interactivity and result visualisation, but the work required to build and validate the models would be extensive : this would require orders of magnitude more work than the spreadsheet described in this paper.

4.3 Potential Enhancements

A full dynamic simulation of climber motion would certainly be part of the engineering of future tether systems, but such a full and precise analysis would be of limited value now while there are many uncertainties surrounding the climber and tether design details.

Some enhancements to model particular design features have already been discussed in Section 3.3.6 : these were removed after evaluation, but could be reinstated if there was a need to revisit those concepts.

Other potential enhancements are as follows.

4.3.1 Accuracy

As discussed in section 4.1, improved spreadsheet accuracy could be achieved by reducing the element height. This would be especially beneficial when multiple climber launches per day were being modeled.

It would also be possible to include an allowance for climber acceleration : at present the calculations assume steady-state vertical motion, with the tractive energy effectively all being converted to potential energy. It would be possible to add a term to reduce this energy transfer by any increase in kinetic energy, but this is unlikely to impact conclusions of comparative studies.

4.3.2 Motion Between GEO and Apex

At present the spreadsheet only calculates climber dynamics for the ascent from Earth to GEO. The journey from GEO to the Apex Anchor is effectively a ‘descent’ as centrifugal force exceeds that of gravity, meaning the climber must continuously brake to maintain a constant speed. The methodology would be similar to that of the ‘ascent’ to GEO, with maximum speed and maximum braking power specified. The effect of the extra outward ‘weight’ of multiple climbers on tether stress would have to be compensated for by a reduction in the Apex Anchor mass or altitude.

It would be necessary to specify a departure velocity from GEO, though it may be best to assume this was the maximum specified velocity and provided by some launch system at GEO or by a small drive motor on the climber. The effective gravity at the Apex Anchor is only -0.054g (at 100,000km altitude), so the braking power would be a small fraction of the drive power needed to ascend from the Earth.

4.3.3 Dashboard Enhancement

The Dashboard could be extended by including tether definition parameters such as those shown in Figure 2, but the value of this is uncertain.

5. CONCLUSIONS

5.1 The spreadsheet methodology described in this paper can automatically yield approximate climber positioning and tether loading for multiple regular launches from the Earth Port to GEO Node. This enables useful comparative studies of climber design and operational parameters, and in particular allows optimisation of daily payloads for a fixed tether loading.

5.2 The spreadsheet output variables of climber potential payload and journey time to GEO have a complex and non-linear dependency on a number of parameters, including maximum allowable tether stress, climber power/mass ratio, climber maximum power, climber power source (solar or continuous), maximum climber speed and solar declination (if solar powered).

5.3 The concept of a solar-powered climber reduces the feasible payload to GEO for a given tether stress limit. A climber with a continuous power supply could carry perhaps 20% more payload to GEO, assuming the same net mass as a solar powered climber. The advantage of continuous power depends on many parameters, including the solar declination (meaning that the time of year is a factor).

5.4 The maximum payload mass launched with a single daily climber departure from the Earth Port will depend on the tether strength and climber design, but if the payload can be separated into smaller packets then a greater daily payload could be raised to GEO for the same tether strength and specific climber performance. Thus a modular climber design would be beneficial : each module would either be used to raise small payloads many times daily, or could be assembled together to raise larger payloads that could not be subdivided.

6. RECOMMENDATIONS

6.1 The spreadsheet should be enhanced and extended to include climber journeys from GEO to the Apex Anchor.

6.2 More work should be undertaken to assess the feasibility of continuous climber power sources. This work might include a ranking exercise comparing on-board power (generation or storage) with transmitted power (wireless or via the tether).

6.3 Future climber design studies should concentrate on smaller modular units with a power output of no more than 1 MW. Design objectives should include maximisation of both the power/mass ratio and the maximum-power speed range.

7. REFERENCES

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